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Focus

Stages of Learning

Manipulatives

- Pattern Blocks and Proportional Thinking
- Introduction to Algebra Blocks
- Algebra Tasks

Other Manipulatives

- Cards
- Dice
- Dominoes
- Letter Chart
- Maps
- Nursery Rhymes
- Sugar Cubes

Discussion

Jerome Bruner (1915 – present)

Bruner is one of the best-known and influential psychologists of the twentieth century. He was one of the key figures in the so-called 'cognitive revolution' - but it is the field of education that his influence has been especially felt. He is one of the key players in constructivism.

A major theme in Bruner’s work is that learning is an active process in which learners construct new ideas or concepts based upon their current/past knowledge. The learner selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so. Cognitive structure (i.e., schema, mental models) provides meaning and organization to experiences and allows the individual to "go beyond the information given".

As far as instruction is concerned, the instructor should try and encourage students to discover principles by themselves. The instructor and student should engage in an active dialog (i.e., Socratic learning). The task of the instructor is to translate information to be learned into a format appropriate to the learner's current state of understanding. Curriculum should be organized in a spiral manner so that the student continually builds upon what they have already learned.

Bruner (1966) states that a theory of instruction should address four major aspects: (1) predisposition towards learning, (2) the ways in which a body of knowledge can be structured so that it can be most readily grasped by the learner, (3) the most effective sequences in which to present material, and (4) the nature and pacing of rewards and punishments. Good methods for structuring knowledge should result in simplifying, generating new propositions, and increasing the manipulation of information.
The ideas originally outlined by Bruner originated from a conference focused on science and math learning. Bruner first illustrated his theory in the context of mathematics and social science programs for young children.

“The concept of prime numbers appears to be more readily grasped when the child, through construction, discovers that certain handfuls of beans cannot be laid out in completed rows and columns. Such quantities have either to be laid out in a single file or in an incomplete row-column design in which there is always one extra or one too few to fill the pattern. These patterns, the child learns, happen to be called prime. It is easy for the child to go from this step to the recognition that a multiple table, so called, is a record sheet of quantities in completed multiple rows and columns. Here is factoring, multiplication and primes in a construction that can be visualized.”

BRUNER’S EDUCATIONAL THEORY

Bruner’s educational theories consist of eight.

1. Theory of Value: What knowledge and skills are worthwhile learning? What are the goals of education?
2. Theory of Knowledge: What is knowledge? How is it different from belief? What is a mistake? What is a lie?

What is knowledge? Knowledge is not simply thinking and the result of intellectual activity and experience, it is the "internalizing of tools that are used within the child's culture" (GB, p. 109). It is characterized by the development of language to convey, in words or symbols, what is felt and known. Language is key to knowledge, it is the primary way that concepts can be taught and questioned. It is also the increasing ability to deal with a variety of activities simultaneously and sequentially (GB, p. I 10). How is knowledge different from belief?. I believe that Bruner would not see much difference between knowledge and belief. He felt that students learn best when their instructor leads them to discovering information on their own. As this is done, knowledge would first occur as a belief, that would later be validated by the instructor. What is a mistake? Because Bruner believes discovery learning, there is, inherently, a trial and error process that the child must go through. Mistakes are therefore simply alternative mental processes and a necessary part of learning. What is a lie? A lie, to Bruner, is anything that takes away from discovery learning, that does not capitalize on young learners who have the ability to learn anything and that does not utilize the technology and tools of our society.

3. Theory of Human Nature: What is a human being? How does it differ from other species? What are the limits of human potential?
4. Theory of Learning: What is learning? How are skills and knowledge acquired?

What is learning? Learning is an active, social process in which students construct new ideas or concepts based on their current knowledge. The student selects the information, forms hypothesis and then integrates this new material
Mat 682 Topics in Mathematics (Fostering Algebraic Thinking)

into his/her own existing knowledge and mental constructs. This is a continual process. Learning occurs in three stages: 1) Enactive- in which children need to experience the concrete (manipulating objects in their hands, touching a real dog) in order to understand. 2) Iconic-students are able to represent materials graphically or mentally (they can do basic addition problems in their heads. 3) Symbolic- students are able to use logic, higher order thinking skills and symbol systems (formulas, such as \( d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \) and understand statements like "too many cooks spoil the broth").

How are skills and knowledge acquired? These things are not acquired gradually, but more in a staircase pattern which consists of spurts and rests. Spurts are caused by certain concepts "clicking", being understood. These "clicks" have to be mastered before others are acquired, before there is movement to the next step. These steps are not linked to age but more toward environment. Environments can slow down the sequence or speed it up. Knowledge was best acquired when students were allowed to discover concepts and information on their own.

5. Theory of Transmission: Who is to teach? By what methods will they teach? What will the curriculum be?

Discovery learning is most important. Interaction between students and their instructors was necessary. What will the curriculum be? "A curriculum should involve the mastery of skills that in turn lead to the mastery of still powerful ones". Bruner believed that curriculum should be organized in a spiral manner, each new concept building on what was previously learned. Mathematics is very important because it is one of the few disciplines, along with poetry, that is able to withstand change. These things, at their foundation, have remained relatively the same for centuries, while the rest of our world has grown and changed.

6. Theory of Society: What is society? What institutions are involved in the educational process?
7. Theory of Opportunity: Who is to be educated? Who is to be schooled?
8. Theory of Consensus: Why do people disagree? How is consensus achieved? Whose opinion takes precedence?

These theories lead to three principles for instruction.

1. Instruction must be concerned with the experiences and contexts that make the student willing and able to learn (readiness).
2. Instruction must be structured, so that students can easily grasp the information being presented (spiral organization).
3. Instruction should be designed to facilitate extrapolation and or fill in the gaps (going beyond the information given).
RECENT WORK

Bruner has come to be critical of the 'cognitive revolution' and has looked to the building of a cultural psychology that takes proper account of the historical and social context of participants. In his 1996 book *The Culture of Education* these arguments were developed with respect to schooling (and education more generally). 'How one conceives of education', he wrote, 'we have finally come to recognize, is a function of how one conceives of the culture and its aims, professed and otherwise' (Bruner 1996: ix-x).

It is surely the case that schooling is only one small part of how a culture inducts the young into its canonical ways. Indeed, schooling may even be at odds with a culture's other ways of inducting the young into the requirements of communal living.... What has become increasingly clear... is that education is not *just* about conventional school matters like curriculum or standards or testing. What we resolve to do in school only makes sense when considered in the broader context of what the society intends to accomplish through its educational investment in the young. How one conceives of education, we have finally come to recognize, is a function of how one conceives of culture and its aims, professed and otherwise. (Jerome S. Bruner 1996: ix-x)

Manipulatives

Manipulatives are tools that can be used to help students to progress from Bruner’s enactive (action) stage of learning to his iconic (pictorial) stage of learning to his symbolic (abstract) stage of learning.

Several virtual manipulatives are available via the Internet. One such site is

http://nlvm.usu.edu/en/nav/vlibrary.html

PATTERN BLOCKS AND PROPORTIONAL THINKING

Pattern blocks are commonly used in the elementary grades to help students to investigate patterns as well as deal with fractions. Ratios and proportions serve as a major component in the development of algebra. Thus, pattern blocks serve as a good first manipulative.

A set of pattern blocks consists of blocks in six geometric, color-coded shapes. The shapes are:

- Green Triangle
- Orange Square
- Blue Rhombus
- Tan Rhombus
- Red Trapezoid
- Yellow Hexagon
The blocks are designed so that the sides fit together so that a tiling will be space filling. The angles are divisors of 360°. This allows the blocks to be used as fractional parts.

- Green Triangle -- \( \frac{1}{6} \)
- Blue Rhombus -- \( \frac{1}{3} \)
- Red Trapezoid -- \( \frac{1}{2} \)
- Yellow Hexagon -- 1

With this small family of fractions, students can explore and gain hands-on experience with comparing fractions, finding equivalent fractions, improper fractions, and modeling addition, subtraction, multiplication and division.

Pattern blocks can help the student to concentrate on fraction algorithms instead of on symbolic manipulation (as illustrated by \( \frac{3}{2} \times \frac{1}{2} \), or \( \frac{1}{3} \div \frac{1}{2} \)).

**Basic Operations**

**Addition**

\[
\frac{1}{3} + \frac{1}{2} = \frac{5}{6}
\]

\[
\frac{1}{6} + \frac{1}{3} = \frac{1}{2}
\]
Subtraction

\[ \frac{1}{2} - \frac{1}{6} = \frac{1}{3} \]

\[ \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \]
Multiplication

\[
\frac{2}{3} \times \frac{1}{6} = \frac{3}{27} = \frac{1}{9}
\]

\[
\frac{1}{2} \times \frac{1}{3} = \frac{3}{18} = \frac{1}{6}
\]

(Note: In the example above, the three trapezoids form a triangle similar to the green triangle. Thus, the proportion \( \frac{3}{18} = \frac{1}{6} \) is true.)
(Note: The square tiles can be used to form grid models multiplication. This allows you to demonstrate denominators other than 6, 3, and 2.)
Division

\[
\frac{3}{2} \div \frac{1}{3} = 4 \frac{1}{2}
\]

\[
\frac{4}{3} \div \frac{1}{6} = 8
\]

**Proportions**

Modeling proportions with pattern blocks can help students see the connection that proportions have to division.
\[
\frac{?}{3} = \frac{3}{2} \\
? \times \frac{1}{3} = \frac{3}{2} \\
? = \frac{3}{2} \div \frac{1}{3}
\]

\[
\frac{?}{6} = \frac{4}{3} \\
? \times \frac{1}{6} = \frac{4}{3} \\
? = \frac{4}{3} \div \frac{1}{6}
\]
INTRODUCTION TO ALGEBRA BLOCKS AND CHIPS

Signed Numbers

The introduction of signed numbers has long been troublesome for some students. (Research shows that two of the biggest areas of difficulty for students as they start to transition from arithmetic to algebra, are fractions and signed numbers.)

A common model used in the primary grades is the chip model. Chips of two different colors are used to represent positives and negatives. A major key to the success of such a model is an understanding of the neutral (zero) concept. Two chips, one of each color, is neutral (zero).
Addition

Addition of signed numbers simply involves combining chips into neutralized pairs and recognizing the remaining chips.

\[ -2 + 3 = ? \]

Subtraction

Subtraction is accomplished by adding the same amount of neutral pairs as the absolute value of the subtrahend.

\[ 5 - (-2) = ? \]
Multiplication

Multiplication can be viewed as repeated addition when one of the multiplicand is positive.

\[ 3 \times (-2) = ? \]

When the signs of both multiplicands are negative, start with as many neutral pairs as is the absolute value of the product of the two numbers (i.e. multiply as if the numbers were positive). Next, interpret the first negative “-” as remove. Experience has shown that students struggle with the multiple meaning of the “-” symbol (minus, negative, opposite, remove, and subtract).

\[ -3 \times (-2) = ? \]

Division

Division of signed numbers is easiest when modeled based off the following definition.

**Definition** If \( a \) and \( b \) are any integers, where \( b \neq 0 \), then \( a \div b \) is the unique number \( c \), if it exists, such that \( a = b \times c \).
\(-6 \div (-3) = ?\)
\[\Rightarrow -3 \times ? = -6\]

Removal of 3 groupings of what will result in \(-6\)?

**Polynomials**

Modeling polynomials can be accomplished with the use of algebra tiles. Algebra tiles are an extension (if you will) of base blocks. Tiles of different dimensions are used to represent different variables and units. There are different manufacturers and each set is slightly different. A template for creating a cardboard set is attached.

Classroom demonstrations and exercises will be based on AlgeBlocks® (by ETA Cuisenaire). A major key to successful use of AlgeBlocks® is an understanding of the neutral (zero) concept. Two tiles, one on each side of an axis, is neutral (zero).
Addition

\[(x + 5) + (x - 3)\]

\[\text{Answer: } 2x + 2\]

\[(x^2 - 5x + 3) + (x^2 + 2x - 1)\]
Answer: $2x^2 - 3x + 2$

Subtraction

$(x + 5) - (2x + 3)$
Answer: \(-x + 8\)

Remove \((x^2 + 5x - 3)\)

\[x^2 + 7x - 2 - (x^2 + 5x - 3)\]
Answer: $2x + 1$

Multiplication

$(x + 5)(x - 2)$
Answer: $x^2 + 3x - 10$

$(x - 3)(x^2 + 3x - 1)$
Answer: $x^3 - 10x + 3$
Division

Division is best understood by understanding multiplication. Think of division as undoing multiplication. You are given the length of one side of the rectangle and asked to rearrange the dividend to form a rectangle.

\[(x^2 + 2x + 1) \div (x + 1)\]

Answer: \(x + 1\)
\[(x^2 - 3x - 10) \div (x - 5)\]

Answer: \(x + 2\)
\[(x^2 + 4x - 5) \div (x - 2)\]
Answer: \((x + 6) \, R \, 7\)

Factoring

GCF

\((2x^2 - 4xy)\)
Answer: 2x

Polynomials

Answer: (x + 2)(x + 3)
Answer: $(x - 5)(x + 3)$

Square Trinomials

$x^2 + 6x + 9$
Answer: \((x + 3)^2\)

\[x^2 - 4x + 4\]
Answer: \((x - 2)^2\)

Linear Equations

Answer: \(x = 7\)
\[3x + 4 = 10\]

Answer: \(x = 2\)
\[4x - 5 = 3x + 2\]

Answer: \(x = 7\)
\[ 4x + 7 = -2x - 5 \]

Answer: \( x = -2 \)
Quadratic Equations

$x^2 + 2x + 1 = 0$

Answer: $(x + 2)(x + 1) = 0$

or

$x = -2$ or $x = -1$

Completing the Square

$x^2 + 4x = 12$
Answer: $(x + 2)^2 = 16$

or

\[ x + 2 = \pm 4 \Rightarrow x = -6 \text{ or } x = 2 \]

\[ x^2 - 4x = 12 \]

Answer: $(x - 2)^2 = 16$
**Other Manipulatives**

Several everyday items can also be used as manipulatives. Several such objects are; cards, dice, dominoes, letter charts, maps, nursery rhymes, and sugar cubes.

Refer to attached activities. The activities are taken from *Algebra: Kindergarten through Grade Nine*.

Wheeling and Dealing (Middle)  
Know it All’s (Middle)  
Cryptograms (Middle)  
The Puzzling U.S. (Middle)  
State the Whole Truth (Middle)  
The Domino Effect (Middle)  
What Goes Where, When? (Middle)  
Cubist Math (Upper)